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Time: 2 Hrs.30 Mins. Marks: 75

N.B. (i) All questions are compulsory.

- (ii) Attempt ANY TWO-sub questions out of Q1, Q2 and Q3.
- (iii) Attempt ANY THREE-sub questions from Q4.
- (iv) Figures to the right indicate marks.
- Q.1. (a) Show that for any two events A and B,  $P(A \cup B) = P(A) + P(B) P(A \cap B).$

Also derive the result when A and B are mutually exclusive events.

- (b) A committee of 3 is to be formed from among 5 boys and 3 girls. What is the probability that the committee shall have
  - i) 2 boys and 1 girl
  - ii) at least one boy
  - iii) more girls than number of boys.
- (c) Two cards are drawn from a normal pack of 52 well-shuffled cards. Find the 10 probability that the cards drawn are
  - i) both black
  - ii) one black and one heart
  - iii) both aces
  - iv) one ace and one king
  - v) both face cards
- (d) In a bolt factory, three machines A, B and C produce 25, 35 and 40 percent of total output respectively and it is found that 5, 4 and 2 percent respectively are defective bolts in their production. If a bolt is chosen at random from the total output, what is the probability that it is defective? If a bolt is chosen and is found to be defective, what is the probability that the bolt came from machine A?
- Q.2. (a) Define Expectation E(X) and Variance V(X) of a discrete random variable 10 X. Show that
  - i) E(aX + b) = a E(X) + b
  - ii)  $V(aX + b) = a^2 V(X)$ , where a and b are constants.
  - (b) Find K in the following case so that p(x) can be regarded as a probability distribution function.

| X :                 | -1         | 0    | 1  | 2                |
|---------------------|------------|------|----|------------------|
| P(X) :              | <u>k+1</u> | 1 13 | k  | $\frac{k-4}{13}$ |
| $\Gamma(\Lambda)$ . | 13         | 13   | 13 | 13               |

Also find Expected value of X.

(c) Following is joint probability mass function of X and Y.

|     | •    |      |      |
|-----|------|------|------|
| x\y | 1    | 2    | 3    |
| 5   |      | 0.05 | 0,10 |
| 10  | 0.15 | 0.20 | 0.20 |
| 15  | 0,10 | 0,05 | 0.05 |
| 20  |      |      | 0.10 |

Obtain- i) Marginal probability distributions of X and Y

- ii) Conditional probability distribution of Y when  $X \ge 3$ .
- iii) Conditional probability distribution of X when  $Y \le 2$

|      | (d) | Explain joint probability distribution of two discrete random variables.  Define (i) joint probability mass function  (ii) marginal probability mass function  (iii) conditional probability mass function.  | 10 |
|------|-----|--|----|
| Q.3. | (a) | Define a random variable X that follows discrete uniform distribution with parameter n. Also find its mean and variance.   | 10 |
|      | (b) | The sales manager of an automobile dealer estimates that 90% of the new cars delivered by them have no defect and so will not be brought back immediately for repair. He sells a fleet of 6 cars to an important customer. What is the probability that —  (i) no car will be brought back | 10 |
|      |     | (ii) all cars will be brought back   |    |
|      |     | (iii) one or more cars will be brought back for repair?  |    |
|      | (c) | A variate X follows Poisson distribution with parameter 5. Evaluate (i) $p(x = 0)$ (ii) $p(x = 1)$ (iii) $p(x \ge 1)$ (iv) $p(x \ne 0)$ . Given that $e^{-5} = 0.00674$ .  | 10 |
|      | (d) | A digit is drawn at random from among the digits $1, 2, 3, 4, 5, 6, 7, 8, 9$ and $0$ . If X denotes the digit drawn, find $p(x)$ , $E(X)$ and $V(X)$ .   | 10 |
| Q.4. | (a) | Tickets numbered from 1 to 100 are well shuffled and a ticket is drawn from it. What is the probability that the selected ticket has:  (i) an odd number  (ii) number 5 or multiple of 5?  | 5  |
|      | (b) | A discrete random variable X takes values -2, 0 and 2 with probabilities 0.2,  | 5  |
| W.   |     | 0.5 and 0.3 respectively. Find the probability distribution function of -  (i) $Y = 2X + 1$ (ii) $Z = X^2 + 1$   |    |
|      | (c) | A Binomial distribution has mean 6 and variance 3. Find n and p.   | 5  |
|      | (d) | On an average three divorce cases are filed in a court of a small city. Find the chance that on a certain day the number of such cases coming up would be (i) one  | 5  |
|      |     | (ii) at least two  |    |
|      |     | (iii) at most two  |    |
|      |     | 1-11/00 to 2-11/1/1/1/1  |    |